Formula 1
Strategy Competition

"One of the most exciting projects I undertook during my final year was working on strategies for a[n imaginary] Formula 1 team. Not only did this experience foster a deep appreciation for Formula 1, but it also exposed me to various applications of mathematics that I hadn't previously considered." (Christiana Antoniou, Masters student at the University of Leeds, 2023)

"Exciting race... one of the only bits of entertainments left in Northern Europe" (Prof. Alan Champneys, Professor of Applied Non-linear Mathematics at the University of Bristol; during the CoViM-19 lockdown, 2020)

"This is more interesting than real F1" (Dr. Ati Sharma, Associate Professor of Aerodynamics at the University of Southampton; 2018)

The competition will take place in three stages: (i) a financial stage, where you will build your racing team by signing drivers and developing your car, (ii) a race strategy stage, where you will decide on the best strategy for each race and (iii) a ten-race championship, which will be livestreamed so you can follow the performance of your team.

1 Financial stage

In the Financial stage of the competition, the team sponsors have provided you with a budget of $8\imath$ (pronouned like the imaginary unit—this is an imaginary competition!). You will be allowed to spend it with a precision of up to one decimal place. Your first decision is to divide this budget into four departments: marketing, chassis development, engine development and reliability. The budget invested into the marketing department will make it more likely for your team to sign the best drivers, while chassis and engine developments directly turn into performance gain and reliability allows the car to be less prone to mechanical failures.

1.1 Marketing

After all the teams have registered for the competition, a list of available drivers will be published. Drivers are ranked according to their driving ability, called Driving and assessed from $0\star$ (worst) to $5\star$ (best). The more you invest in marketing, the more attractive your team looks, but do not forget that this is a competitive market: your competitors will do the same.

Let m_i be the marketing investment of team i and n is the total number of competing teams, any given driver will sign with team i with probability:

$$p_i = \frac{m_i^h}{\displaystyle\sum_{j=1}^{n} m_j^h}, \tag{1}$$

where $h = 3.5$.

1.2 Chassis and engine development

Besides completing your driver lineup, you need to invest the rest of your budget in your car. There are two components to the car: the chassis and the engine. Some circuits favour cars with a strong chassis while some others favour those with a powerful engine. The performance level of your chassis and of your engine (expressed in $\star$) will be equal to the amount of $\imath$ that you invest in each of these components.

1.3 Reliability

Mechanical failure is part of the race deciding factors. Although one has to accept it, there exist ways to maximize reliability by hiring a more experienced mechanical crew, building/purchasing higher quality mechanical parts, etc. The reliability of your car directly determinates the likelihood of retiring from a race. The retirement probability for a driver at a given rate is given by:

$$p_{\text{DNF}} = \frac{\left[1 - \operatorname{erf}\left(0.75 * r - 1.5\right)\right]^2}{4}, \tag{2}$$

where r is the amount in $\imath$ invested in car reliability and DNF stands for "Did Not Finish", an acronym often used in motorsports.

1.4 Example

Team A, Team B and Team C have $8.0\imath$ of budget and there are 6 drivers on the market: Archibald (Driving: $4.8\star$), Beulah (Driving: $3.9\star$), Cecil (Driving: $3.0\star$), Dagobert (Driving: $2.1\star$), Earl (Driving: $1.0\star$) and Fritz (Driving: $0.2\star$). We consider equation (1) with $h = 2$ in this example for simplicity.

Team A wants to build a team around good drivers and decides to spend $4.0\imath$ on Marketing, $0.8\imath$ on Chassis, $3.0\imath$ on Engine and $0.2\imath$ on Reliability. Team B wants consistency and opts for a balanced

Team	Chassis	Engine	Reliability	Driver	Driving	Performance	p_{DNF}
Team A	0.8⋆	3.0⋆	0.2ı	Archibald	4.8⋆	8.6⋆	0.94
				Cecil	3.0⋆	6.8⋆	
Team B	3.0⋆	2.0⋆	2.0ı	Beulah	3.9⋆	8.9⋆	0.11
				Dagobert	2.1⋆	7.1⋆	
Team C	4.0⋆	4.0⋆	0.0ı	Earl	1.0⋆	9.0⋆	0.97
				Fritz	0.2⋆	8.2⋆	

Table 1: Summary of the example team lineups and strengths for a balanced circuit.

strategy, investing $1.0ı$ in Marketing, $3.0ı$ in Chassis, $2.0ı$ in Engine and $2.0ı$ in Reliability. Team C wants the fastest car and, thus, does neither invest in Marketing nor in Reliability, but spends $4.0ı$ in Chassis and $4.0ı$ in Engine.

The signing of each driver is done in order from the strongest (highest Driving) to the weakest (lowest Driving). For a given driver, a number x_{RNG} between 0 and 1 is drawn from a uniform distribution. If $x_{RNG} \leq p_a$, where p_a is the probability associated with Team A, then the driver signs with Team A. If, however, $p_a < x_{RNG} \leq p_a + p_b$, where p_b is the probability associated with Team B, they sign with Team B. Lastly, if $p_a + p_b < x_{RNG} \leq p_a + p_b + p_c = 1$, where p_c is the probability associated with Team C, they sign with Team C. Given the teams' Marketing investments, drivers have probability p_a to be attracted to Team A, p_b to be attracted to Team B and p_c to be attracted to Team C, where:

$$p_a = \frac{16}{17}, \qquad p_b = \frac{1}{17}, \qquad p_c = 0. \tag{3}$$

Archibald draws 0.258 and signs with Team A. Then, Beulah draws 0.994 and signs with Team B, followed by Cecil who signs with Team A after drawing 0.620. At this stage, Team A has a full lineup and does no longer need drivers. Remain Team B and Team C with updated probabilities:

$$p_b = 1, \qquad p_c = 0, \tag{4}$$

calculated in the same way as before but without the contribution from Team A. Dagobert draws 0.137 and signs with Team B. Only Team C has free seats, so Earl and Fritz take them.

The team strengths for a balanced circuit[1] are summarized in table 1.

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[1]The meaning of *balanced circuit* will become clear in Section 2.

2 Race Strategy

You will have to provide a strategy for each race and each driver. A strategy consists in the number of pit-stops, the moment at which they are planned and the type of tyres to put on. To devise your strategy, you will need to know the circuit specifics: how lap times vary, how long pit-stops last and some additional rules which are explained below.

2.1 Circuit data

Each circuit is presented on "cards" similar to that in Figure 1. The first line indicates that Monaco will be the third race in the championship. The race will consist in 78 laps and the associated base time is $t_b = 74$s. On the second line, we can see that the circuit favours drivers (see Section 2.2) and that the Overtake delta is 1.0s, meaning that to overtake, one must be at least 1.0s per lap faster than the preceding driver. The Overtake delta also defines the Follow delta, the latter being half of the former and meaning that one cannot follow another car closer than a gap of 0.5s (see Section 2.7). The third line indicates the fuel load penalty as a function of the number of laps already raced in the stint, l, and the total number of laps planned in the stint l_{max} (see Section 2.2). That line also gives the approximative time that making a pit-stop costs as a function of the number of laps planned in the next stint l_{max}^+ (see Section 2.3 for the complete expression, including additive randomness). The following lines on Figure 1 indicate the tyre wear laws for each of the tyre compounds: soft, medium and hard (see Section 2.2).
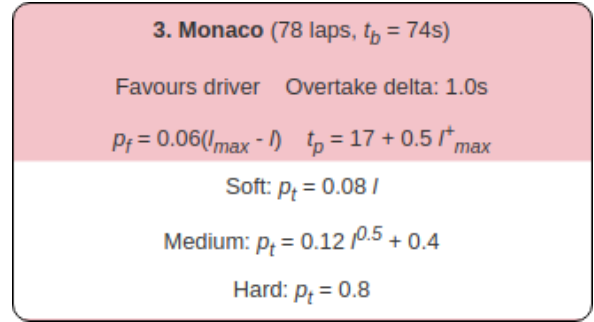

Figure 1: Example of circuit data.

2.2 Lap time calculation

The laptime t is calculated in the following way:

$$t = t_b + t_{\text{perf}} + p_t + p_f + R_{\text{lap}}, \tag{5}$$

where t_b is the base time which only depends on the circuit, t_{perf} is the base performance proportional to the number of $\star$ of your driver given by:

$$t_{\text{perf}} = -0.15 * (\text{Performance} + \text{Setup}), \tag{6}$$

where Setup is used to simulate the (random) effect that free practice sessions have on the performance of the car and where Performance takes into account the strength of your driver and car in relation with the circuit.

To calculate Setup, a random number is drawn from a uniform distribution between -0.5 and 0.5 for each team. Following this, another such number is drawn for each driver. Setup is the sum of these variables and is expressed in $\star$. It takes values between $-1\star$ and $1\star$, ensuring that drivers from the same team do not have Setup values more than $1\star$ apart.

Furthermore,

$$\text{Performance} = \frac{3 \left(c_d \, \text{Driving} + c_c \, \text{Chassis} + c_e \, \text{Engine} \right)}{c_d + c_c + c_e}, \tag{7}$$

where $c_d = 1$, $c_c = 1$ and $c_e = 1$ on balanced circuits. Certain circuits favour particular components of your car. When this is the case, the associated coefficient is doubled. For example, Belgium is a circuit

that favours chassis and engine, so $c_d = 1$, $c_c = 2$ and $c_e = 2$. Assuming Setup=0, the drivers of Table 1 have the following Performance value: Archibald (Team A) 7.44⋆, Cecil (Team A) 6.36⋆, Beulah (Team B) 8.34⋆, Dagobert (Team B) 7.26⋆, Earl (Team C) 10.20⋆ and Fritz (Team C) 9.72⋆.

Lastly, p_t (resp. p_f) is the tyre wear (resp. fuel load) penalty specified in the circuit list, and R_{lap} is a random function computed for each lap and each driver that reflects realistic pace irregularities from the driver:

$$R_{lap} = 4 * (x_{RNG} - 0.4)^3 + 0.3 * x_{RNG}, \tag{8}$$

where x_{RNG} is a random number drawn using the uniform distribution between 0 and 1.

2.3 Pit-stops

The more you race, the more your tyres wear and lose in performance. There are three different tyre compounds with different tyre wear laws per race: soft, medium and hard tyres. These tyres change from one circuit to another and are specified in the circuit list. Furthermore, the more fuel onboard, the heavier and, thus, the slower the car. The remedy to this is to stop to change the tyres and refuel your car, which also allows you to start the race with a lighter car.

During a pit-stop, your pit crew changes your tyres as well as refuels your car. You lose time in two different ways: (i) by getting into the pit lane where driving is much slower than on the circuit, and (ii) during the action of the mechanics. Changing tyres typically takes a couple of seconds (Formula 1 mechanics are highly skilled and coordinated!) and it takes 1s for them to pump enough fuel into the car for 2 laps of autonomy. As a result, a pit-stop costs:

$$t_p = t_{pb} + 0.5 l_{max}^+ + R_{pit}, \tag{9}$$

where t_{pb} is the circuit-specific time wasted due to pit lane driving, l_{max}^+ represents the number of laps of fuel to transfer into the car (or stint length) and $R_{pit} = 2 * x_{RNG}$ is a random function reflecting the variability of the mechanics performance and of the time lost in the pit lane. Here again, x_{RNG} is a random number drawn using the uniform distribution between 0 and 1. The pit-stop law is specified without R_{pit} in the circuit list for simplicity.

2.4 Mechanical failure

The risk of mechanical failure is explained in Section 1.3. At the start of a race and for each driver, a random number is drawn from a uniform distribution between 0 and 1. If this number is smaller than the driver's p_{DNF}, they will retire before the end of the race. The lap number at which the driver retires is subsequently drawn with each lap being equally probable.

2.5 Qualifying lap

To decide the starting grid for the race, all the drivers have to go through one qualifying lap. The order in which they do so is random and the lap is calculated assuming the best possible conditions (fresh tyres and minimal fuel load).

At the end of the qualifications, the starting grid is filled from the driver who set the fastest qualifying lap to the one with the slowest qualifying lap. To simulate the gaps created by the starting grid, the race time of the poleman is initialized at 0s and the race time of each subsequent driver is initialized at a value that is larger than that of the preceding driver by the Follow delta (see Section 2.1).

Each driver starts the race on a brand new set of tyres of the same compound as the tyres used during the qualifications and with the fuel load specified in your strategy. As a result, choose your first stint tyres carefully as they impact your qualifications.
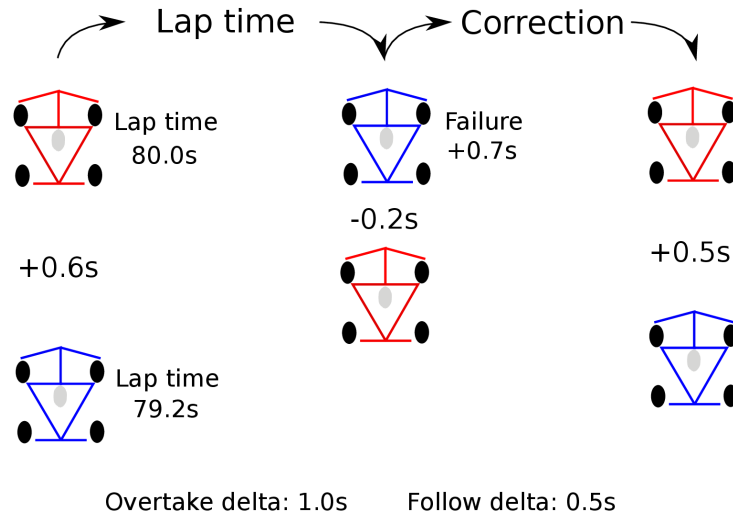
Figure 2: Illustration of an overtaking attempt with an Overtake delta of 1.0s and a Follow delta of 0.5s. Driver Red led driver Blue by 0.6s and produced a lap time of 80.0s against 79.2s for Blue. This placed Blue ahead of Red by 0.2s after taking the lap times into account, so an overtaking manoeuver has been attempted. Blue's race pace is only 0.8s faster than Red's, which is below the Overtake delta and, thus, insufficient for the overtaking to be successful. Blue is then penalized to end up behind Red by a time equal to the Follow delta. In this instance, the penalty is 0.7s.

2.6   Drag Reduction System

Any driver within less than 1s of their preceding driver on a given lap is allowed to use, during this lap only, his Drag Reduction System (DRS). When used, this device improves the car performance by 0.3s per lap. Using DRS is not allowed during the first two laps of a race.

2.7   Traffic

Due to physical constraints and human latency, it is impossible to follow a car infinitely closely. To model this, cars are not allowed to follow more closely than a time gap called the Follow delta (see Section 2.1). If a car ends up closer to the car ahead, it is considered to be slowed down by traffic (due to dirty air, for instance) and its lap time is penalized in such a way that a gap of at least the Follow delta is restored between both cars.

Overtaking takes more than just being faster: it requires having a comfortable pace advantage compared to the target driver. During the race, this is modelled by comparing the race pace of the drivers involved in an overtaking attempt. An overtaking attempt is detected if there is a change in racing order after adding the current lap time to the race time for all drivers. When this is the case, the race pace (last lap time) of the driver attempting the overtake is compared with that of the defending driver. If the race pace difference is greater than a quantity called the Overtake delta (see Section 2.1), then the overtake is successful. If it is not, a time penalty is applied to the driver attempting the overtake in such a way as to place them back behind the other involved driver with a gap at least equal to the Follow delta.

An example of overtaking attempt, illustrating both the overtaking and following rules, is presented in figure 2. In practice, whenever a penalty is applied to place a driver at a certain gap behind another, an additional penalty is applied equal to a random number drawn from a uniform distribution between 0s and 0.1s to avoid odd-looking intervals between drivers.

3 Competition rules

3.1 Important information

All interaction will be made via emailing: `c.m.l.beaume@leeds.ac.uk.` Please do not hesitate to send an email if you have any question or need further clarification. The webpage of the competition is: `http://cbeaume.com/en/teaching_f1.html.`

3.2 Timeline

Registration: You first need to provide your team name before the deadline indicated on the webpage by sending an email to the above address and including your team name. Team names are restricted to 15 characters and are to be respectful. Here is an example of valid registration email:

> Title: Team A registration
>
> Dear Dr. Beaume,
>
> I would like to register team "Team A" for the competition.
>
> Thank you,
>
> Cédric

The driver list will be published shortly after the end of the registration period.

Financial stage: You will need to indicate in another email to the same address before before the deadline indicated on the webpage how many $\imath$ you want to spend in marketing (you are allowed to spend non-integer values of $\imath$—only one decimal place is allowed), in chassis development, engine development and in reliability. Here is an example of valid financial strategy based on an initial budget of $10\imath$:

> Title: Team A financial strategy
>
> Dear Dr. Beaume,
>
> The financial strategy of "Team A" is as follows:
> Marketing: $4.6\imath$
> Chassis: $1.1\imath$
> Engine: $0.9\imath$
> Reliability: $3.4\imath$
>
> Thank you,
>
> Cédric

More details on this stage are given in Section 1. The driver signing event will be livestreamed shortly after the deadline.

Race strategy stage: Your race strategy for a given race has to be submitted at least one week before the corresponding race. Details on this stage are given in Section 2 and the circuit list is given at

`http://cbeaume.com/en/teaching_f1.html`. Please send the strategies in plain text (do not send spreadsheets or attachments). To submit your strategy, you need to specify the track, the driver, their starting tyres (on which they will qualify and do their first stint), followed by the list of pit-stops, each being constituted of the lap number at which the pit-stop is to be observed and the type of tyres to put on. Lastly, each strategy should finish by the number of laps of the given race. Here is an example of submission from Team A for Australia, the first race of the season:

> Title: Team A strategy for Australia
>
> Dear Dr. Beaume,
>
> Here is the strategy of Team A for Australia:
>
> 1. Australia
> Archibald   S 10 M 30 H 60
> Cecil   H 29 H 60
>
> Thank you,
>
> Cédric

You can send race strategies for multiple races within the same email. Please respect the format shown above including the title of the email containing both the team name and the race(s) covered! Failure to do so might result in an unintentionally faulty implementation of your strategy. In this example and for the Australian Grand Prix, Archibald would qualify on soft tyres and race with them until lap 10, where he would pit for medium tyres, race 20 laps with them and pit on lap 30 for hard tyres which he would keep until the end of the race. His team mate, Cecil, would start on hard tyres and pit on lap 29 to get a new set of hard tyres and go till the end of the race with them.

Ten race championship: Races will be livestreamed at the dates mentioned on the webpage.

3.3   Championship classification

The championship consists in 10 races on different circuits. The list of the circuit with their race number is provided at `http://www.cbeaume.com/en/teaching_f1.html`. At the end of each race, drivers and teams score points if they finished the race as a function of their position. The number of point scoring drivers depends on the number of drivers registered for the championship:

- Less than or 12 drivers: 10pts, 6pts, 4pts, 3pts, 2pts, 1pt.

- 13–16 drivers: 10pts, 8pts, 6pts, 5pts, 4pts, 3pts, 2pts, 1pt.

- 17–20 drivers: 20pts, 16pts, 13pts, 10pts, 8pts, 6pts, 4pts, 3pts, 2pts, 1pt.

- More than 20 drivers: 25pts, 20pts, 16pts, 13pts, 11pts, 10pts, 9pts, 8pts, 7pts, 6pts, 5pts, 4pts, 3pts, 2pts, 1pt.

Driver championship tie-breakers will be made by comparing the best race result, then the second bests result, etc.

The number of points a team scores is the sum of the points of the drivers racing for the team. Team championship tie-breakers will be made by comparing drivers results in a similar way as for the driver championship tie-breakers.

3.4 Penalties

The following is a list of penalties that can be applied during the competition. The organizer can apply other penalties if he identifies situations that are in breach of the competition rules or that lead to unfair advantages. These would set a precedent and will be added here.

3.4.1 Registration

Penalty 0.1 – Registration delay: If a team's registration has not reached the organizer by the registration deadline, a penalty will be applied in the form of a budget reduction by $0.1\imath$ per day after the deadline. The maximum such penalty is $0.6\imath$.

Penalty 0.2 – Late entry: If a team's registration takes place 14 or more days after the registration deadline, Penalty 0.1 will be applied together with the withdrawal of the team from the driver signing event. The team will be allocated two drivers with $0.0\star$ driving skill.

Penalty 0.3 – Invalid team name: If a competitor provides an unacceptable team name, whether it be offensive, or not respect the given format, $0.3\imath$ will be taken off their budget. This penalty will be applied once the registration deadline is reached.

3.4.2 Financial stage

Penalty 1.1 – Budget delay: If a budget is not provided before the deadline, a penalty will be applied in the form of a budget reduction by $0.2\imath$ per day after the deadline. The maximum such penalty is $1.2\imath$. This penalty will be applied equally to the reliability, chassis and engine budgets with any leftover applied to the reliability budget, then to the marketing budget if the reliability budget reaches $0.0\imath$. In case one budget goes to $0.0\imath$ during the application of the penalty, the penalty application continues disregarding this budget. If a team's budget has not been submitted before the day of the driver signing event, the marketing budget of the associated team is set to $0.0\imath$.

Penalty 1.2 – Invalid budget: If an invalid budget is submitted, it will not be considered and might lead to application Penalty 1.1.

3.4.3 Race strategy

Penalty 2.1 – Strategy submission delay: Submission of a strategy after the race strategy deadline will lead to the application, at the end of the race, of a race time penalty for both drivers equal to 5s per day beyond the deadline. The maximum such penalty per race per driver is 30s.

Penalty 2.2 – Strategy error: If a team submits a strategy containing a typo or minor error, a default value will replace it and no penalty will be applied. Fixing a strategy after the deadline will lead to application of Penalty 2.1.

Penalty 2.3 – Strategy missing: If a team did not submit your strategy before race day, a default, computer-generated (usually mediocre) strategy will be used instead and Penalty 2.1 will be applied.

Penalty 2.4 – Double-stacking: If a team pits its two drivers on the same lap, the second driver will be subject to a race time penalty equal to t_{pb} at the end of the race. This penalty is not cumulative.

3.4.4 Bots

Definition: A bot is a team that is not (i.e., no longer) managed by an individual. The term bot is also used as an adjective.

Bot rule 1 – Bot declared before financial strategy submission: If a bot is declared before submission of the related team's financial strategy, Penalty 1.1 will be applied, resulting in the bot's marketing budget being set to $0.0i$ and in its total budget being $1.2i$ lower than that of other teams. Furthermore, the bot's budget will be applied nearly equally between chassis, engine and reliability, as described in Penalty 1.1.

Bot rule 2 – Bot declared before race strategy submission: If a bot is declared before the related team's race strategy is submitted, the bot's strategy will be replaced by an automated one based on competing teams strategies. The bot drivers will always use hard tyres. If two or more non-bot drivers chose a zero or one pit-stop strategy, the bot drivers will use a one pit-stop strategy, otherwise, the bot drivers will use a two-stop strategy. Bot one-stop stragegies consist in pitting on the laps closest to 55% of the race distance, with the first bot driver pitting first. Bot two-stop strategies consist in pitting on the laps closest to 40% and 70% of the race distance, with the first bot driver pitting first.

4 Livestream

The races will be livestreamed on Youtube (link and times available on the webpage). They will unfold in the following way: (i) a qualifying round, where the drivers will drive one by one to produce one timed lap, and (ii) the race.

The race will unfold lap by lap, at the pace dictated by the race leader: if they drive lap 20 in 84s, the livestream will freeze after lap 19 for 84s before displaying lap 20. An example of race display is shown in Figure 3. The first three columns represent the rank, driver and team. The fourth column is the gap


```
        Classification after 63/78 laps

                                  Gap      Last    Tyres              Tyres used
 1 Allan JOHNSON        HV22          +  0.000   73.797   15H              SMH
 2 Alfredo ALCARRIA     BURP          + 23.884   73.442    2S              HHS
 3 Jooris GOOTJES       F1enberg      + 27.600   73.068*   1S              HSSS
 4 David WILKINSON      BURP          + 33.426   73.607    1S              HSSS
 5 Cyprien MONNEREAU    Senna'matic   + 35.448   74.582   12H              SSSH
 6 Mark WINKELMANN      forkBOMB      + 38.735   74.687    7M              HMM
 7 Luigi DISANTI        Gotta Go Fast + 39.272   73.856    1S   Traffic   HMSS
 8 Pavel ONDRA          Fisher Racing + 46.960   74.312   10M              HSM
 9 Ricardo RAUL         forkBOMB      + 54.731   74.068    2S              SMSS
10 Carlos ARIZAGA       BlueFlag Racing + 66.899 75.076    5M              SMMM
11 Philipp KRAFFT       Senna'matic   + 71.734   74.660   46H              SH
12 Simao FARIA          Jose Elmez    + 72.722   99.149    0S DRS          MMMS
13 Clayton SWIFT        BlueFlag Racing + 76.982 75.438   12M              MMM
14 Alexander KOLTSOV    F1enberg      --------   -------   DNF
15 Aaron KILPATRICK     Jose Elmez    --------   -------   DNF
16 Felix WOLLEN         HV22          --------   -------   DNF
17 Udo KRALLINGER       Gotta Go Fast --------   -------   DNF
18 Michel FABRON        Fisher Racing --------   -------   DNF
```

Figure 3: Example of livestream display.

to the leader, followed by the last lap time in column 5. The fastest last lap is indicated with an asterisk following the lap time. Both column 4 and 5 will display a series of dashes if the driver retired. Column 6 displays the tyres being used by the driver as well as their age. Column 7 (resp. 8) might display DRS (resp. Traffic) in case the driver used the Drag Reduction System (resp. got slowed down by traffic) during their last lap. The last column shows the history of the tyres used during the race. For example, in figure 3, Luigi Disanti, from Gotta Go Fast, has raced his 63rd lap (out of 78) and is in 7th position, 39.272s away from the leader. His last lap was done in 73.856s and he is on soft tyres, which have only completed 1 lap so far. He did not use DRS on his last lap but was slowed down by Mark Winkelmann and has closed down on the latter sufficiently to be able to use DRS next lap. So far, Disanti use hard tyres, followed by medium tyres and two sets of soft tyres.